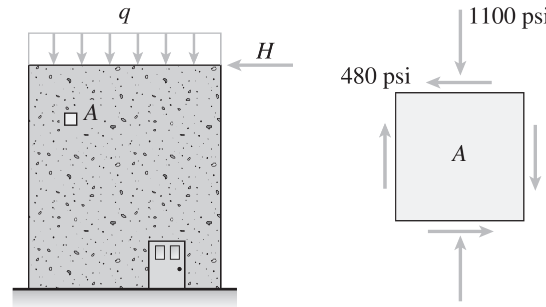
Problem 7.3-9 A shear wall in a reinforced concrete building is subjected to a vertical uniform load of intensity q and a horizontal force H, as shown in the first part of the figure. (The force H represents the effects of wind and earthquake loads.) As a consequence of these loads, the stresses at point A on the surface of the wall have the values shown in the second part of the figure (compressive stress equal to 1100 psi and shear stress equal to 480 psi).

(a) Determine the principal stresses and show them on a sketch of a properly oriented element.

(b) Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.



Solution 7.3-9 Shear wall

$\sigma_x = 0 \quad \sigma_y = -1100 \text{ psi} \quad \tau_{xy} = -480 \text{ psi}$

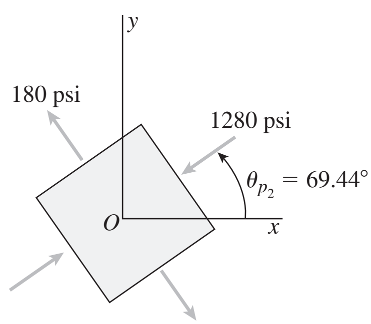
(a) Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.87273$$

$2\theta_p = -41.11°$ and $\theta_p = -20.56°$
$2\theta_p = 138.89°$ and $\theta_p = 69.44°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -41.11°$: $\sigma_{x_1} = 180$ psi
For $2\theta_p = 138.89°$: $\sigma_{x_1} = -1280$ psi
Therefore, $\sigma_1 = 180$ psi and $\theta_{p_1} = -20.56°$
$\sigma_2 = -1280$ psi and $\theta_{p_2} = 69.44°$ ←

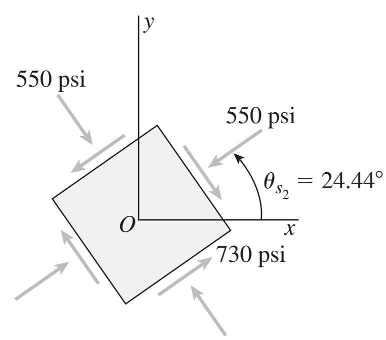


(b) Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 730 \text{ psi}$$

$\theta_{s_1} = \theta_{p_1} - 45° = -65.56°$ and $\tau = 730$ psi
$\theta_{s_2} = \theta_{p_1} + 45° = 24.44°$ and $\tau = -730$ psi ←

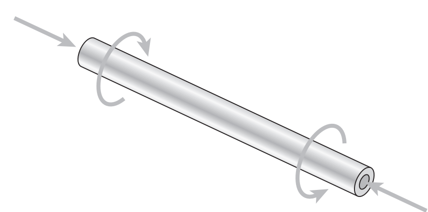
$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -550 \text{ psi} \leftarrow$$

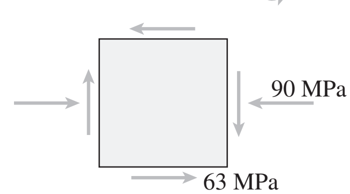


Problem 7.3-10 A propeller shaft subjected to combined torsion and axial thrust is designed to resist a shear stress of 63 MPa and a compressive stress of 90 MPa (see figure).

(a) Determine the principal stresses and show them on a sketch of a properly oriented element.

(b) Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.





Solution 7.3-10 **Propeller shaft**

$\sigma_x = -90$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = -63$ MPa

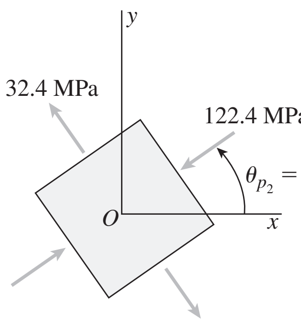
(a) PRINCIPAL STRESSES

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 1.4000$$

$2\theta_p = 54.46°$ and $\theta_p = 27.23°$
$2\theta_p = 234.46°$ and $\theta_p = 117.23°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 54.46°$: $\sigma_{x_1} = -122.4$ MPa
For $2\theta_p = 234.46°$: $\sigma_{x_1} = 32.4$ MPa



Therefore,

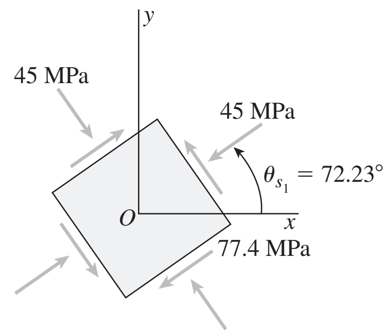
$\sigma_1 = 32.4$ MPa and $\theta_{p_1} = 117.23°$
$\sigma_2 = -122.4$ MPa and $\theta_{p_2} = 27.23°$ ←

(b) MAXIMUM SHEAR STRESSES

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 77.4 \text{ MPa}$$

$\theta_{s_1} = \theta_{p_1} - 45° = 72.23°$ and $\tau = 77.4$ MPa
$\theta_{s_2} = \theta_{p_1} + 45° = 162.23°$ and $\tau = -77.4$ MPa ←

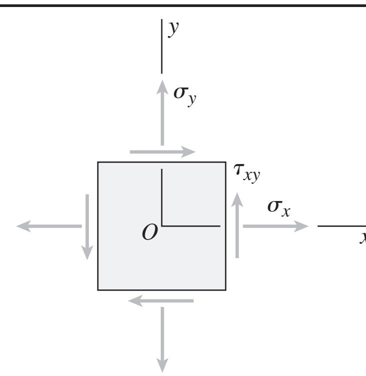
$$\sigma_{\text{aver}} = \frac{\sigma_x + \sigma_y}{2} = -45 \text{ MPa} \quad ←$$



Problems 7.3-11 through 7.3-16 An element in *plane stress* (see figure) is subjected to stresses σ_x, σ_y, and τ_{xy}.

(a) Determine the principal stresses and show them on a sketch of a properly oriented element.

(b) Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.



Data for 7.3-11 $\sigma_x = 3500$ psi, $\sigma_y = 1120$ psi, $\tau_{xy} = -1200$ psi

Solution 7.3-11 **Plane stress**

$\sigma_x = 3500$ psi $\quad \sigma_y = 1120$ psi $\quad \tau_{xy} = -1200$ psi

(a) PRINCIPAL STRESSES

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1.0084$$

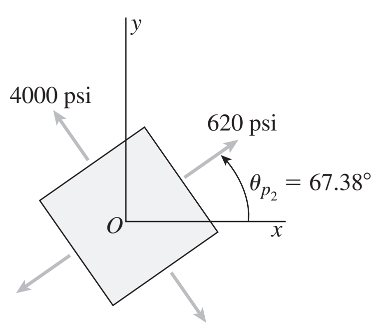
$2\theta_p = -45.24°$ and $\theta_p = -22.62°$
$2\theta_p = 134.76°$ and $\theta_p = 67.38°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -45.24°$: $\sigma_{x_1} = 4000$ psi
For $2\theta_p = 134.76°$: $\sigma_{x_1} = 620$ psi

Therefore,

$\sigma_1 = 4000$ psi and $\theta_{p_1} = -22.62°$
$\sigma_2 = 620$ psi and $\theta_{p_2} = 67.38°$ ←

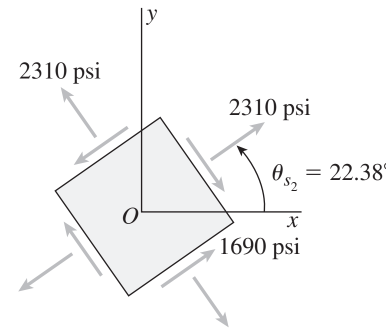


(b) MAXIMUM SHEAR STRESSES

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 1690 \text{ psi}$$

$$\left.\begin{array}{l}\theta_{s_1} = \theta_{p_1} - 45° = -67.62° \text{ and } \tau = 1690 \text{ psi} \\ \theta_{s_2} = \theta_{p_1} + 45° = 22.38° \text{ and } \tau = -1690 \text{ psi}\end{array}\right\} \leftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = 2310 \text{ psi} \leftarrow$$



Data for 7.3-12 $\sigma_x = 2100$ kPa, $\sigma_y = 300$ kPa, $\tau_{xy} = -560$ kPa

Solution 7.3-12 **Plane stress**

$\sigma_x = 2100$ kPa $\quad \sigma_y = 300$ kPa $\quad \tau_{xy} = -560$ kPa

(a) PRINCIPAL STRESSES

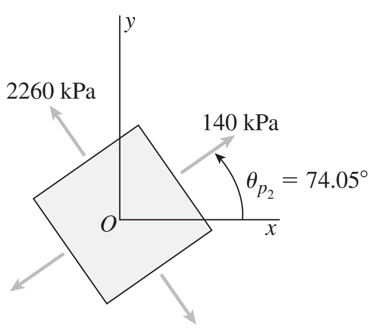
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.6222$$

$2\theta_p = -31.89°$ and $\theta_p = -15.95°$
$2\theta_p = 148.11°$ and $\theta_p = 74.05°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -31.89°$: $\sigma_{x_1} = 2260$ kPa
For $2\theta_p = 148.11°$: $\sigma_{x_1} = 140$ kPa

$$\left.\begin{array}{l}\text{Therefore, } \sigma_1 = 2260 \text{ kPa and } \theta_{p_1} = -15.95° \\ \sigma_2 = 140 \text{ kPa and } \theta_{p_2} = 74.05°\end{array}\right\} \leftarrow$$

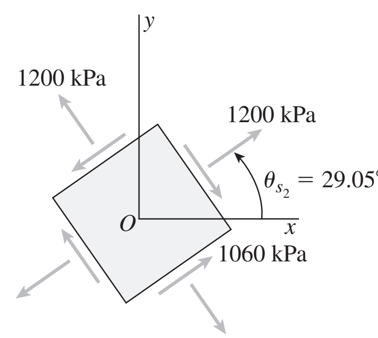


(b) MAXIMUM SHEAR STRESSES

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 1060 \text{ kPa}$$

$$\left.\begin{array}{l}\theta_{s_1} = \theta_{p_1} - 45° = -60.95° \text{ and } \tau = 1060 \text{ kPa} \\ \theta_{s_2} = \theta_{p_1} + 45° = 29.05° \text{ and } \tau = -1060 \text{ kPa}\end{array}\right\} \leftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = 1200 \text{ kPa} \leftarrow$$



Data for 7.3-13 $\sigma_x = 15{,}000$ psi, $\sigma_y = 1{,}000$ psi, $\tau_{xy} = 2{,}400$ psi

Solution 7.3-13 Plane stress

$\sigma_x = 15{,}000$ psi $\quad \sigma_y = 1{,}000$ psi $\quad \tau_{xy} = 2{,}400$ psi

(a) Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 0.34286$$

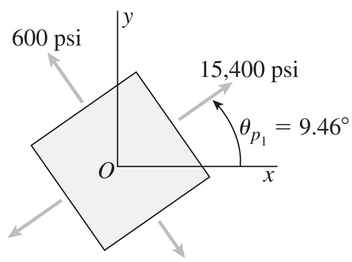
$2\theta_p = 18.92°$ and $\theta_p = 9.46°$
$2\theta_p = 198.92°$ and $\theta_p = 99.46°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 18.92°$: $\quad \sigma_{x_1} = 15{,}400$ psi

For $2\theta_p = 198.92°$: $\quad \sigma_{x_1} = 600$ psi

Therefore, $\sigma_1 = 15{,}400$ psi and $\theta_{p_1} = 9.46°$
$\sigma_2 = 600$ psi and $\theta_{p_2} = 99.96°$ ←

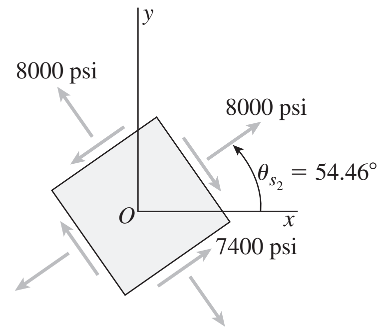


(b) Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7{,}400 \text{ psi}$$

$\theta_{s_1} = \theta_{p_1} - 45° = -35.54°$ and $\tau = 7{,}400$ psi
$\theta_{s_2} = \theta_{p_1} + 45° = 54.46°$ and $\tau = -7{,}400$ psi ←

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = 8{,}000 \text{ psi} \leftarrow$$



Data for 7.3-14 $\sigma_x = 16$ MPa, $\sigma_y = -96$ MPa, $\tau_{xy} = -42$ MPa

Solution 7.3-14 Plane stress

$\sigma_x = 16$ MPa $\quad \sigma_y = -96$ MPa $\quad \tau_{xy} = -42$ MPa

(a) Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.7500$$

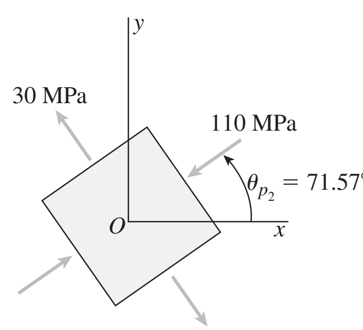
$2\theta_p = -36.87°$ and $\theta_p = -18.43°$
$2\theta_p = 143.13°$ and $\theta_p = 71.57°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -36.87°$: $\quad \sigma_{x_1} = 30$ MPa

For $2\theta_p = 143.13°$: $\quad \sigma_{x_1} = -110$ MPa

Therefore, $\sigma_1 = 30$ MPa and $\theta_{p_1} = -18.43°$
$\sigma_2 = -110$ MPa and $\theta_{p_2} = 71.57°$ ←

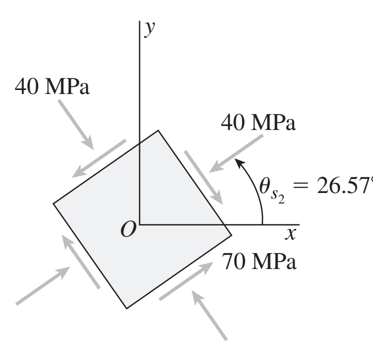


(b) Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 70 \text{ MPa}$$

$\theta_{s_1} = \theta_{p_1} - 45° = -63.43°$ and $\tau = 70$ MPa
$\theta_{s_2} = \theta_{p_1} + 45° = 26.57°$ and $\tau = -70$ MPa ←

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa} \leftarrow$$



Data for 7.3-15 $\sigma_x = -3000$ psi, $\sigma_y = -12{,}000$ psi, $\tau_{xy} = 6000$ psi

Solution 7.3-15 Plane stress

$\sigma_x = -3000$ psi $\quad \sigma_y = -12{,}000$ psi
$\tau_{xy} = 6000$ psi

(a) Principal stresses

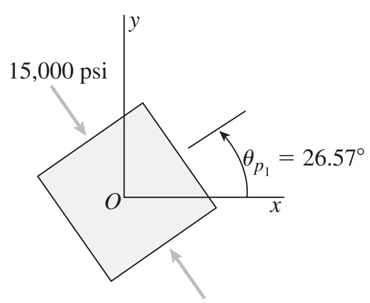
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 1.3333$$

$2\theta_p = 53.13°$ and $\theta_p = 26.57°$
$2\theta_p = 233.13°$ and $\theta_p = 116.57°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 53.13°$: $\quad \sigma_{x_1} = 0$
For $2\theta_p = 233.13°$: $\quad \sigma_{x_1} = -15{,}000$ psi
Therefore,

$$\left.\begin{array}{l}\sigma_1 = 0 \text{ and } \theta_{p_1} = 26.57° \\ \sigma_2 = -15{,}000 \text{ psi and } \theta_{p_2} = 116.57°\end{array}\right\} \longleftarrow$$

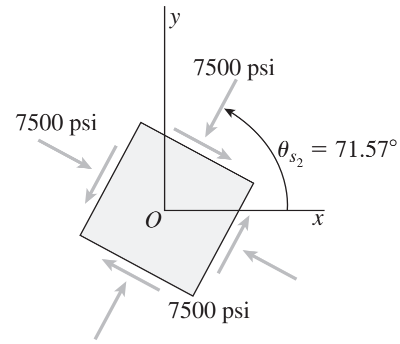


(b) Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7500 \text{ psi}$$

$$\left.\begin{array}{l}\theta_{s_1} = \theta_{p_1} - 45° = -18.43° \text{ and } \tau = 7500 \text{ psi} \\ \theta_{s_2} = \theta_{p_1} + 45° = 71.57° \text{ and } \tau = -7500 \text{ psi}\end{array}\right\} \longleftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -7500 \text{ psi} \longleftarrow$$



Data for 7.3-16 $\sigma_x = -100$ MPa, $\sigma_y = 50$ MPa, $\tau_{xy} = -50$ MPa

Solution 7.3-16 Plane stress

$\sigma_x = -100$ MPa $\quad \sigma_y = 50$ MPa $\quad \tau_{xy} = -50$ MPa

(a) Principal stresses

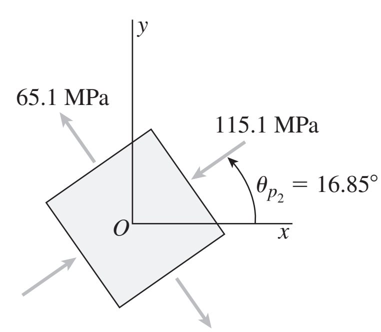
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 0.66667$$

$2\theta_p = 33.69°$ and $\theta_p = 16.85°$
$2\theta_p = 213.69°$ and $\theta_p = 106.85°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 33.69°$: $\quad \sigma_{x_1} = -115.1$ MPa
For $2\theta_p = 213.69°$: $\sigma_{x_1} = 65.1$ MPa
Therefore,

$$\left.\begin{array}{l}\sigma_1 = 65.1 \text{ MPa and } \theta_{p_1} = 106.85° \\ \sigma_2 = -115.1 \text{ MPa and } \theta_{p_2} = 16.85°\end{array}\right\} \longleftarrow$$

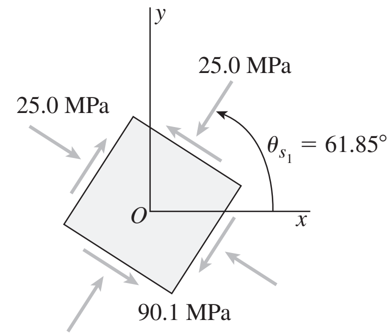


(b) Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 90.1 \text{ MPa}$$

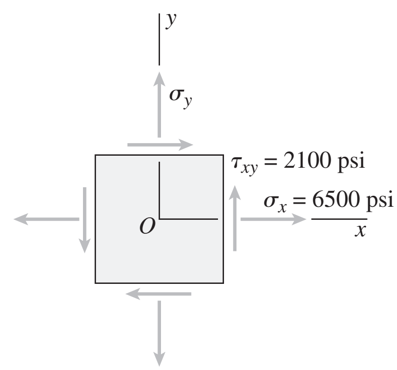
$$\left.\begin{aligned}\theta_{s_1} &= \theta_{p_1} - 45° = 61.85° \text{ and } \tau = 90.1 \text{ MPa}\\ \theta_{s_2} &= \theta_{p_1} + 45° = 151.85° \text{ and } \tau = -90.1 \text{ MPa}\end{aligned}\right\} \longleftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -25.0 \text{ MPa} \longleftarrow$$



Problem 7.3-17 At a point on the surface of a machine component the stresses acting on the x face of a stress element are $\sigma_x = 6500$ psi and $\tau_{xy} = 2100$ psi (see figure).

What is the allowable range of values for the stress σ_y if the maximum shear stress is limited to $\tau_0 = 2900$ psi?



Solution 7.3-17 **Allowable range of values**

$\sigma_x = 6500$ psi $\quad \tau_{xy} = 2100$ psi $\quad \sigma_y = ?$

Find the allowable range of values for σ_y if the maximum allowable shear stresses is $\tau_0 = 2900$ psi.

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \text{Eq. (1)}$$

or

$$\tau_{max}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \qquad \text{Eq. (2)}$$

Solve for σ_y

$$\sigma_y = \sigma_x \pm 2\sqrt{\tau_{max}^2 - \tau_{xy}^2}$$

Substitute numerical values:

$$\sigma_y = 6500 \text{ psi} \pm 2\sqrt{(2900 \text{ psi})^2 - (2100 \text{ psi})^2}$$

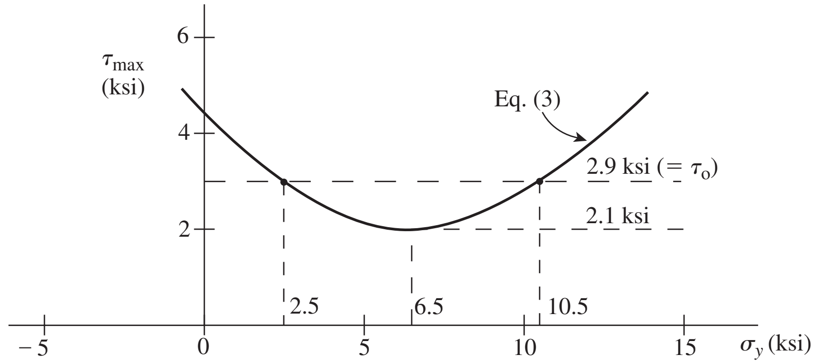
$$= 6500 \text{ psi} \pm 4000 \text{ psi}$$

Therefore, $2500 \text{ psi} \leq \sigma_y \leq 10{,}500 \text{ psi}$ $\longleftarrow$

Graph of τ_{max}

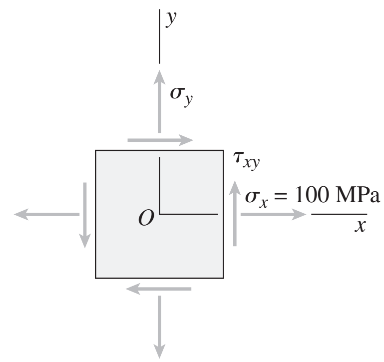
From Eq. (1):

$$\tau_{max} = \sqrt{\left(\frac{6500 - \sigma_y}{2}\right)^2 + (2100)^2} \qquad \text{Eq. (3)}$$



Problem 7.3-18 At a point on the surface of a machine component the stresses acting on the x face of a stress element are $\sigma_x = 45$ MPa and $\tau_{xy} = 30$ MPa (see figure).

What is the allowable range of values for the stress σ_y if the maximum shear stress is limited to $\tau_0 = 34$ MPa?



Solution 7.3-18 Allowable range of values

$\sigma_x = 45$ MPa $\quad \tau_{xy} = 30$ MPa $\quad \sigma_y = ?$

Find the allowable range of values for σ_y if the maximum allowable shear stresses is $\tau_0 = 34$ MPa.

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \text{Eq. (1)}$$

or

$$\tau_{max}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \qquad \text{Eq. (2)}$$

Solve for σ_y

$$\sigma_y = \sigma_x \pm 2\sqrt{\tau_{max}^2 - \tau_{xy}^2}$$

Substitute numerical values:

$$\sigma_y = 45 \text{ MPa} \pm 2\sqrt{(34 \text{ MPa})^2 - (30 \text{ MPa})^2}$$

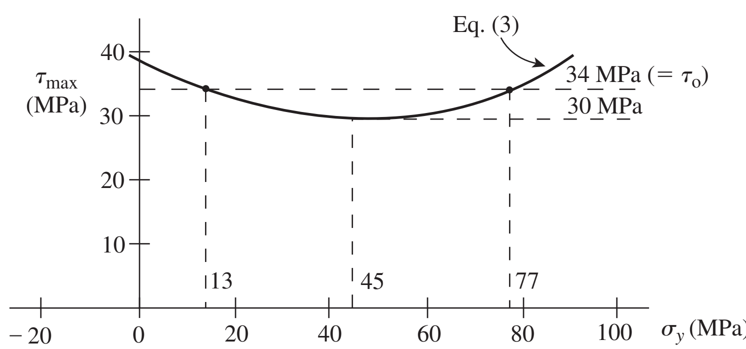
$$= 45 \text{ MPa} \pm 32 \text{ MPa}$$

Therefore, $13 \text{ MPa} \le \sigma_y \le 77 \text{ MPa}$ ←

Graph of τ_{max}

From Eq. (1):

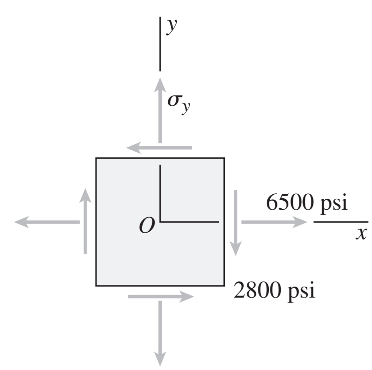
$$\tau_{max} = \sqrt{\left(\frac{45 - \sigma_y}{2}\right)^2 + (30)^2} \qquad \text{Eq. (3)}$$



Problem 7.3-19 An element in *plane stress* is subjected to stresses $\sigma_x = 6500$ psi and $\tau_{xy} = -2800$ psi (see figure). It is known that one of the principal stresses equals 7300 psi in tension.

(a) Determine the stress σ_y.

(b) Determine the other principal stress and the orientation of the principal planes; then show the principal stresses on a sketch of a properly oriented element.



Solution 7.3-19 **Plane stress**

$\sigma_x = 6500$ psi $\tau_{xy} = -2800$ psi $\sigma_y = ?$
One principal stress = 7300 psi (tension)

(a) STRESS σ_y

Because σ_x is smaller than the given principal stress, we know that the given stress is the larger principal stress.

$\sigma_1 = 7300$ psi

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substitute numerical values and solve for σ_y:

$\sigma_y = -2500$ psi ←

(b) PRINCIPAL STRESSES

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.62222$$

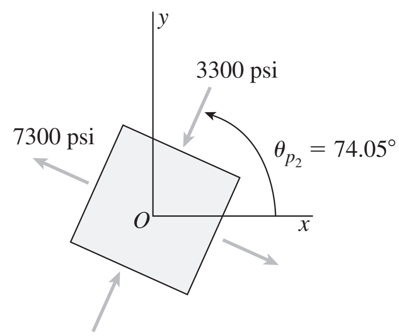
$2\theta_p = -31.891°$ and $\theta_p = -15.945°$
$2\theta_p = 148.109°$ and $\theta_p = 74.053°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -31.891°$: $\sigma_{x_1} = 7300$ psi
For $2\theta_p = 148.109°$: $\sigma_{x_1} = -3300$ psi

Therefore,

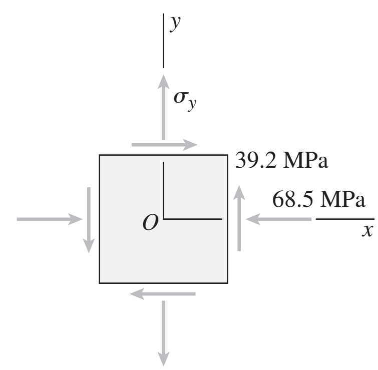
$\sigma_1 = 7300$ psi and $\theta_{p_1} = -15.95°$
$\sigma_2 = -3300$ psi and $\theta_{p_2} = 74.05°$ ←



Problem 7.3-20 An element in *plane stress* is subjected to stresses $\sigma_x = -68.5$ MPa and $\tau_{xy} = 39.2$ MPa (see figure). It is known that one of the principal stresses equals 26.3 MPa in tension.

(a) Determine the stress σ_y.

(b) Determine the other principal stress and the orientation of the principal planes; then show the principal stresses on a sketch of a properly oriented element.



Solution 7.3-20 **Plane stress**

$\sigma_x = -68.5$ MPa $\tau_{xy} = 39.2$ MPa $\sigma_y = ?$
One principal stress = 26.3 MPa (tension)

(a) STRESS σ_y

Because σ_x is smaller than the given principal stress, we know that the given stress is the larger principal stress.

$\sigma_1 = 26.3$ MPa

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substitute numerical values and solve for σ_y:

$\sigma_y = 10.1$ MPa ←

(b) PRINCIPAL STRESSES

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.99746$$

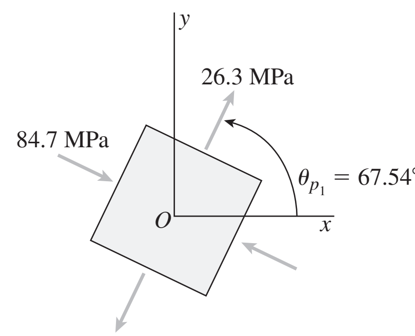
$2\theta_p = -44.93°$ and $\theta_p = -22.46°$
$2\theta_p = 135.07°$ and $\theta_p = 67.54°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -44.93°$: $\sigma_{x_1} = -84.7$ MPa
For $2\theta_p = 135.07°$: $\sigma_{x_1} = 26.3$ MPa

Therefore,

$\sigma_1 = 26.3$ MPa and $\theta_{p_1} = 67.54°$
$\sigma_2 = -84.7$ MPa and $\theta_{p_2} = -22.46°$ ←

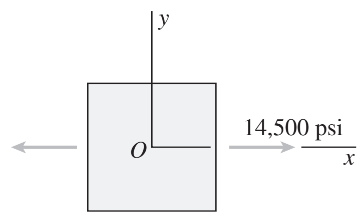


Mohr's Circle for Plane Stress

The problems for Section 7.4 are to be solved using Mohr's circle. Consider only the in-plane stresses (the stresses in the xy *plane).*

Problem 7.4-1 An element in *uniaxial stress* is subjected to tensile stresses $\sigma_x = 14{,}500$ psi, as shown in the figure.

Using Mohr's circle, determine (a) the stresses acting on an element oriented at a counterclockwise angle $\theta = 24°$ from the x axis and (b) the maximum shear stresses and associated normal stresses. Show all results on sketches of properly oriented elements.



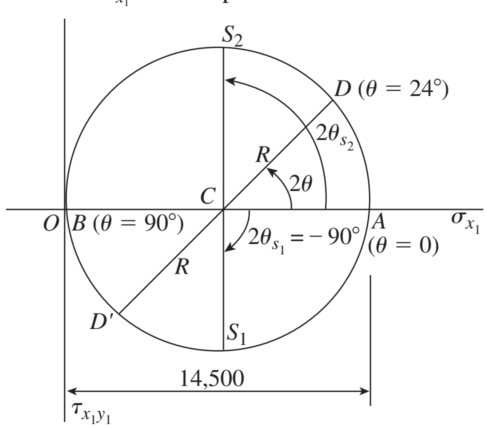
Solution 7.4-1 **Uniaxial stress**

$\sigma_x = 14{,}500$ psi $\quad \sigma_y = 0 \quad \tau_{xy} = 0$

(a) ELEMENT AT $\theta = 24°$ (All stresses in psi)

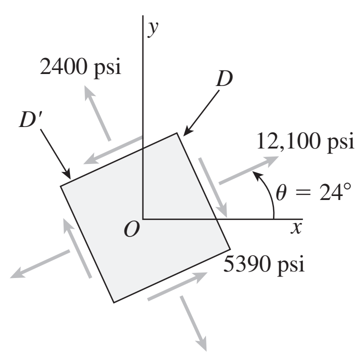
$2\theta = 48° \quad \theta = 24° \quad R = 7250$ psi

Point C: $\sigma_{x_1} = 7250$ psi



Point D: $\sigma_{x_1} = R + R\cos 2\theta = 12{,}100$ psi
$\tau_{x_1y_1} = R\sin 2\theta = -5390$ psi

Point D': $\sigma_{x_1} = R - R\cos 2\theta = 2400$ psi
$\tau_{x_1y_1} = 5390$ psi



(b) MAXIMUM SHEAR STRESSES

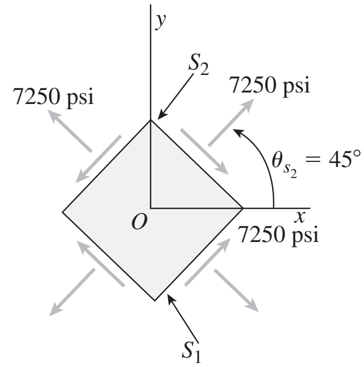
Point S_1: $2\theta_{s_1} = -90°$ $\quad \theta_{s_1} = -45°$

$\tau_{max} = R = 7250 \text{ psi}$

Point S_2: $2\theta_{s_2} = 90°$ $\quad \theta_{s_2} = 45°$

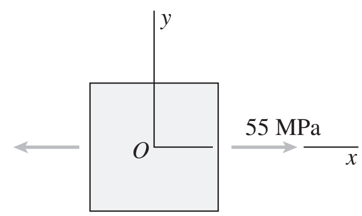
$\tau_{min} = -R = -7250 \text{ psi}$

$\sigma_{aver} = R = 7250 \text{ psi}$



Problem 7.4-2 An element in *uniaxial stress* is subjected to tensile stresses $\sigma_x = 55$ MPa, as shown in the figure.

Using Mohr's circle, determine (a) the stresses acting on an element oriented at an angle $\theta = -30°$ from the x axis (minus means clockwise) and (b) the maximum shear stresses and associated normal stresses. Show all results on sketches of properly oriented elements.



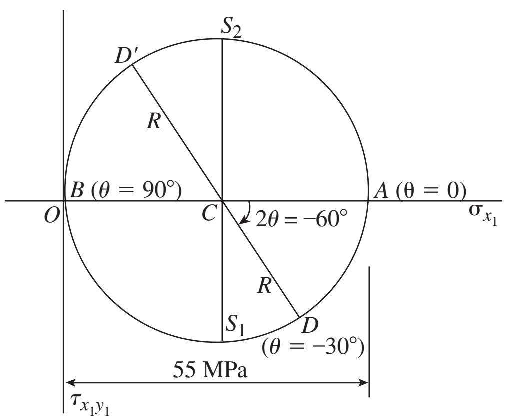
Solution 7.4-2 Uniaxial stress

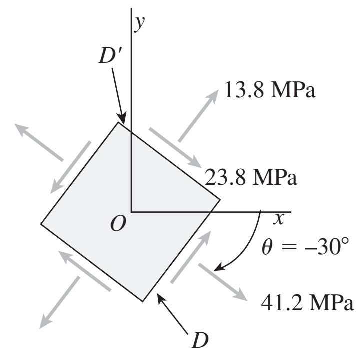
$\sigma_x = 55 \text{ MPa} \quad \sigma_y = 0 \quad \tau_{xy} = 0$

(a) ELEMENT AT $\theta = -30°$ (All stresses in MPa)

$2\theta = -60° \quad \theta = -30° \quad R = 27.5 \text{ MPa}$

Point C: $\sigma_{x_1} = 27.5 \text{ MPa}$





Point D: $\sigma_{x_1} = R + R\cos|2\theta|$

$= R(1 + \cos 60°) = 41.2 \text{ MPa}$

$\tau_{x_1y_1} = R\sin|2\theta| = R\sin 60° = 23.8 \text{ MPa}$

Point D': $\sigma_{x_1} = R - R\cos|2\theta| = 13.8 \text{ MPa}$

$\tau_{x_1y_1} = -R\sin|2\theta| = -23.8 \text{ MPa}$

(b) MAXIMUM SHEAR STRESSES

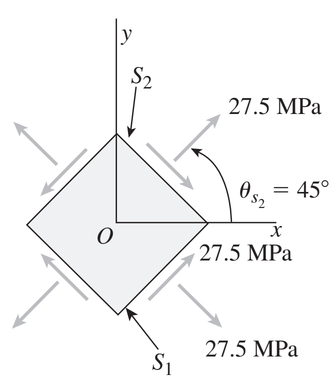
Point S_1: $2\theta_{s_1} = -90°$ $\quad \theta_{s_1} = -45°$

$\tau_{max} = R = 27.5 \text{ MPa}$

Point S_2: $2\theta_{s_2} = 90°$ $\quad \theta_{s_2} = 45°$

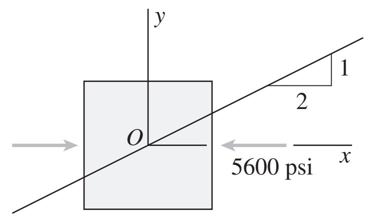
$\tau_{min} = -R = -27.5 \text{ MPa}$

$\sigma_{aver} = R = 27.5 \text{ MPa}$



Problem 7.4-3 An element in *uniaxial stress* is subjected to compressive stresses of magnitude 5600 psi, as shown in the figure.

Using Mohr's circle, determine (a) the stresses acting on an element oriented at a slope of 1 on 2 (see figure) and (b) the maximum shear stresses and associated normal stresses. Show all results on sketches of properly oriented elements.



Solution 7.4-3 Uniaxial stress

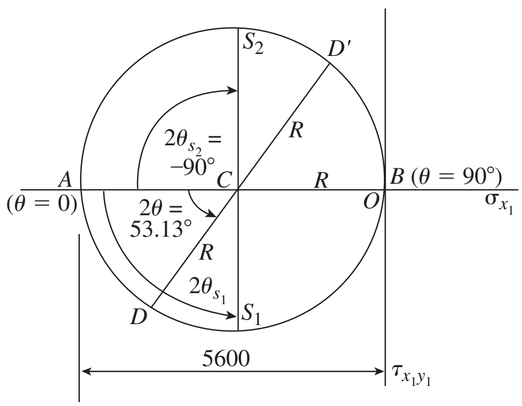
$\sigma_x = -5600$ psi $\quad \sigma_y = 0 \quad \tau_{xy} = 0$

(a) ELEMENT AT A SLOPE OF 1 ON 2

(All stresses in psi) $\quad \theta = \arctan \frac{1}{2} = 26.565°$

$$2\theta = 53.130° \qquad \theta = 26.57° \qquad R = 2800 \text{ psi}$$

Point C: $\sigma_{x_1} = -2800$ psi

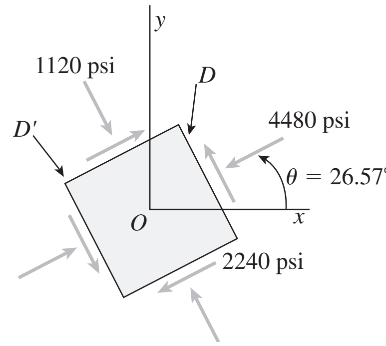


Point D: $\sigma_{x_1} = -R - R\cos 2\theta = -4480$ psi

$\tau_{x_1y_1} = R \sin 2\theta = 2240$ psi

Point D': $\sigma_{x_1} = -R + R\cos 2\theta = -1120$ psi

$\tau_{x_1y_1} = -R \sin 2\theta = -2240$ psi



(b) MAXIMUM SHEAR STRESSES

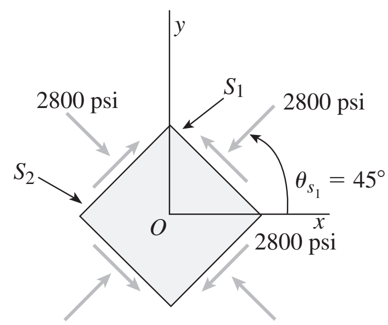
Point S_1: $2\theta_{s_1} = 90° \qquad \theta_{s_1} = 45°$

$\tau_{max} = R = 2800$ psi

Point S_2: $2\theta_{s_2} = -90° \qquad \theta_{s_2} = -45°$

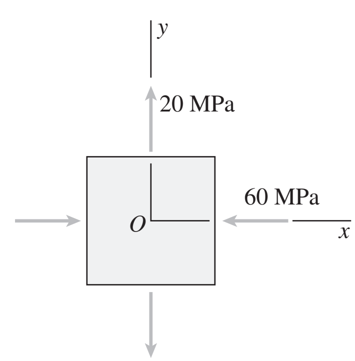
$\tau_{min} = -R = -2800$ psi

$\sigma_{aver} = -R = -2800$ psi



Problem 7.4-4 An element in *biaxial stress* is subjected to stresses $\sigma_x = -60$ MPa and $\sigma_y = 20$ MPa, as shown in the figure.

Using Mohr's circle, determine (a) the stresses acting on an element oriented at a counterclockwise angle $\theta = 22.5°$ from the x axis and (b) the maximum shear stresses and associated normal stresses. Show all results on sketches of properly oriented elements.



Solution 7.4-4 **Biaxial stress**

$\sigma_x = -60$ MPa $\quad \sigma_y = 20$ MPa $\quad \tau_{xy} = 0$

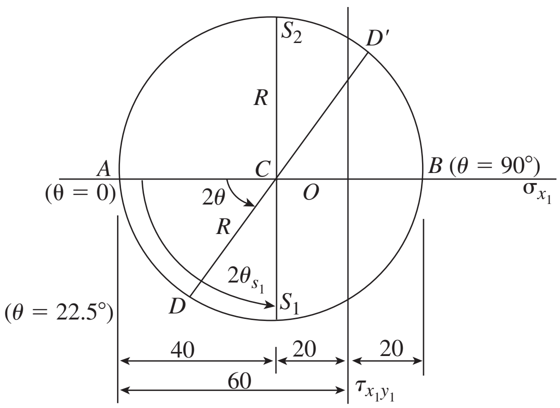
(a) ELEMENT AT $\theta = 22.5°$

(All stresses in MPa)

$2\theta = 45° \quad \theta = 22.5°$

$2R = 60 + 20 = 80$ MPa $\quad R = 40$ MPa

Point C: $\sigma_{x_1} = -20$ MPa

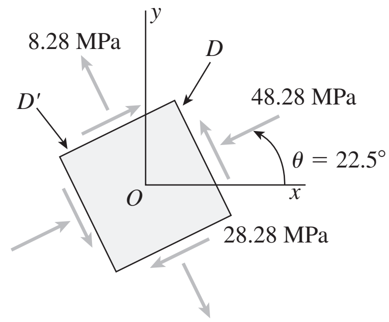


Point D: $\sigma_{x_1} = -20 - R\cos 2\theta = -48.28$ MPa

$\tau_{x_1y_1} = R\sin 2\theta = 28.28$ MPa

Point D': $\sigma_{x_1} = R\cos 2\theta - 20 = 8.28$ MPa

$\tau_{x_1y_1} = -R\sin 2\theta = -28.28$ MPa



(b) MAXIMUM SHEAR STRESSES

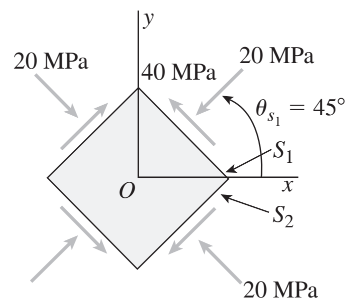
Point S_1: $2\theta_{s_1} = 90° \quad \theta_{s_1} = 45°$

$\tau_{max} = R = 40$ MPa

Point S_2: $2\theta_{s_2} = -90° \quad \theta_{s_2} = -45°$

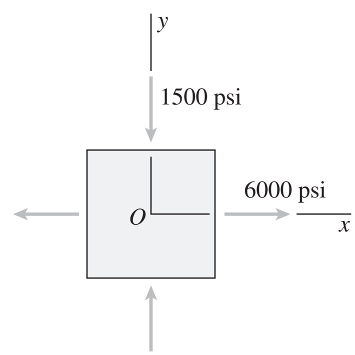
$\tau_{min} = -R = -40$ MPa

$\sigma_{aver} = -20$ MPa



Problem 7.4-5 An element in *biaxial stress* is subjected to stresses $\sigma_x = 6000$ psi and $\sigma_y = -1500$ psi, as shown in the figure.

Using Mohr's circle, determine (a) the stresses acting on an element oriented at a counterclockwise angle $\theta = 60°$ from the x axis and (b) the maximum shear stresses and associated normal stresses. Show all results on sketches of properly oriented elements.



Solution 7.4-5 **Biaxial stress**

$\sigma_x = 6000$ psi $\quad \sigma_y = -1500$ psi $\quad \tau_{xy} = 0$

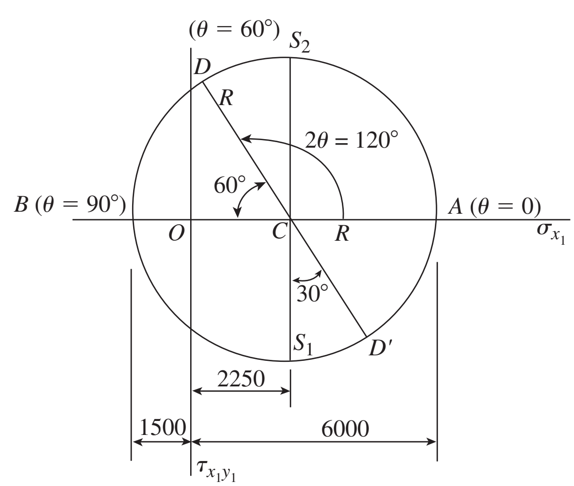
(a) ELEMENT AT $\theta = 60°$

(All stresses in psi)

$2\theta = 120° \quad \theta = 60°$

$2R = 7500$ psi $\quad R = 3750$ psi

Point C: $\sigma_{x_1} = 2250$ psi

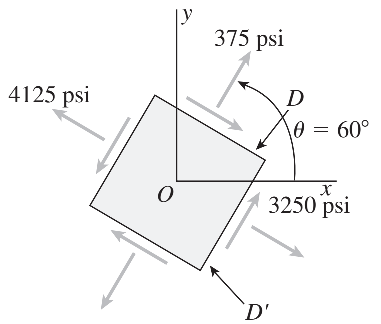


Point D: $\sigma_{x_1} = 2250 - R\cos 60° = 375$ psi

$\tau_{x_1y_1} = -R\sin 60° = -3248$ psi

Point D': $\sigma_{x_1} = 2250 + R\cos 60° = 4125$ psi

$\tau_{x_1y_1} = R\sin 60° = 3248$ psi



(b) MAXIMUM SHEAR STRESSES

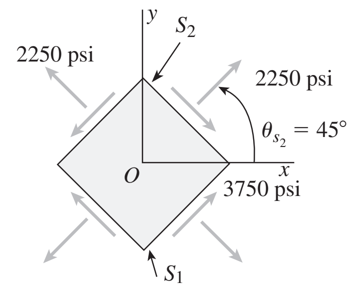
Point S_1: $2\theta_{s_1} = -90°$ $\theta_{s_1} = -45°$

$\tau_{max} = R = 3750$ psi

Point S_2: $2\theta_{s_2} = 90°$ $\theta_{s_2} = 45°$

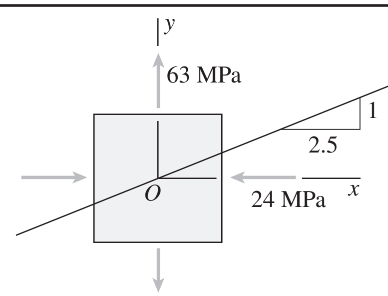
$\tau_{min} = -R = -3750$ psi

$\sigma_{aver} = 2250$ psi



Problem 7.4-6 An element in *biaxial stress* is subjected to stresses $\sigma_x = -24$ MPa and $\sigma_y = 63$ MPa, as shown in the figure.

Using Mohr's circle, determine (a) the stresses acting on an element oriented at a slope of 1 on 2.5 (see figure) and (b) the maximum shear stresses and associated normal stresses. Show all results on sketches of properly oriented elements.

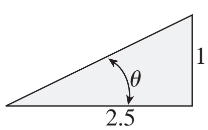


Solution 7.4-6 Biaxial stress

$\sigma_x = -24$ MPa $\quad \sigma_y = 63$ MPa $\quad \tau_{xy} = 0$

(a) ELEMENT AT A SLOPE OF 1 ON 2.5

(All stresses in MPa) $\theta = \arctan \dfrac{1}{2.5} = 21.801°$



$2\theta = 43.603°$

$\theta = 21.801°$

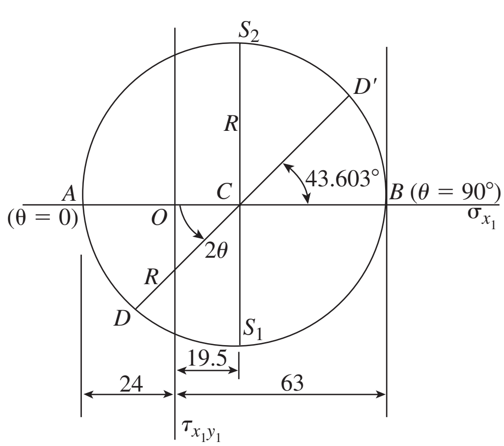
$2R = 87$ MPa

$R = 43.5$ MPa

Point C: $\sigma_{x_1} = 19.5$ MPa

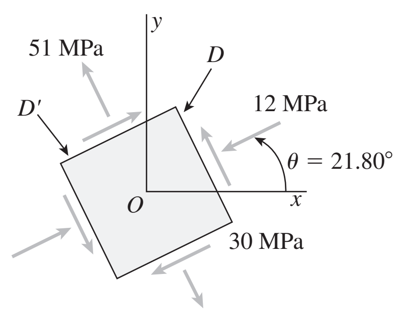
Point D: $\sigma_{x_1} = -R\cos 2\theta + 19.5 = -12$ MPa

$\tau_{x_1y_1} = R\sin 2\theta = 30$ MPa



Point D': $\sigma_{x_1} = 19.5 + R\cos 2\theta = 51$ MPa

$\tau_{x_1y_1} = -R\sin 2\theta = -30$ MPa



(b) MAXIMUM SHEAR STRESSES

Point S_1: $2\theta_{s_1} = 90°$ $\theta_{s_1} = 45°$

$\tau_{max} = R = 43.5$ MPa

Point S_2: $2\theta_{s_2} = -90°$ $\theta_{s_2} = -45°$

$\tau_{min} = -R = -43.5$ MPa

$\sigma_{aver} = 19.5$ MPa